F–1308

Sub. Code
7MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define subspace of a vector space with an example.

2. Define a basis of v.

3. What is meant by annihilator of w?

4. Define an inner product space.

5. What is the degree of $\sqrt{2}+\sqrt{3}$ over Q?

6. Define a splitting field over F.

7. Define a fixed field of a group of automorphisms of k with an example.

8. What is meant by the Galois group of $f(x)$?

9. Define right and left invertible element of $A(V)$.

10. Define the following terms:

    (a) Hermitian; (b) Normal

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the intersection of two subspaces of v is a subspace of v.

Or

(b) Prove that $F^{(n)}$ is isomorphic $F^{(m)}$ if and only if $n = m$.

12. (a) With the usual notations, prove that $A(A(W)) = W$.

Or

(b) If V is a finite-dimensional inner product space and W is a subspace of V then prove that $(W^{\perp})^{\perp} = W$.

13. (a) State and prove the Remainder theorem.

Or

(b) If F is of characteristic 0 and $f(x) \in F[x]$ is such that $f'(x) = 0$, prove that $f(x) = \alpha \in F$.

14. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) If $\alpha_1, \alpha_2, \alpha_3$ are the roots of the cubic polynomial $x^3 + 7x^2 - 8x + 3$, find the cubic polynomial whose roots are $\alpha_1^3, \alpha_2^3, \alpha_3^3$.

15. (a) If V is finite-dimensional over F, then prove that $T \in A(V)$ is invertible if and only the constant term of the minimal polynomial for T is not 0.

Or

(b) If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) If F is the field of real numbers, prove that the vectors $(1,1,0,0)$, $(0,1,-1,0)$, and $(0,0,0,3)$ in $F^{(4)}$ are linearly independent over F.

    (b) If V is finite-dimensional and T is an isomorphism of V into V, prove that T must map V onto V.

17. Let V be a finite-dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, $[L:F]=[L:K][K:F]$.

19. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

20. Let $V = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of $T \in A(V)$ in the basis $v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (0,0,1)$. Find the matrix of T in the basis $u_1 = (1,1,0)$, $u_2 = (1,2,0)$, $u_3 = (1,2,1)$.

F–1310

Sub. Code
7MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Show that the direction cosines of the tangent at the point (x, y, z) to the cone $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

2. Write down the Pfaffian differential equation.

3. Eliminate the arbitrary function f from the equation $z = f(x - y)$.

4. Write down the general solution of the linear partial differential equation $p_p + Q_q = R$.

5. Find the complete integral of the equation $p + q = pq$.

6. When will you say that a first order partial differential is separable?

7. Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ is satisfied by $z = \frac{1}{x}\phi(x-y) + \phi'(y-x)$, where ϕ is an arbitrary function.

8. Find the particular integral of the equation $(D^2 - D^1)z = 2y - x^2$.

9. Define exterior Neumann problem.

10. State the one–dimensional diffusion equation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equations

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

Or

(b) If $\vec{x}$ is a vector such that $\vec{x} \cdot curl\, \vec{x} = 0$ and μ is an arbitrary function of x, y, z then prove that $(\mu \vec{x}) \cdot curl(\mu \vec{x}) = 0$.

12. (a) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

Or

(b) Find the characteristic of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0, y^2 = z$.

13. (a) Find the condition that two partial differential equations are compatible.

Or

(b) Solve $p^2x + q^2y = z$ using Charpit's method.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

Or

(b) Derive the solution of the equation :

$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} = 0$ for the region $r \geq 0, z \geq 0$

satisfying the conditions :

(i) $V \to 0$ and $Z \to \infty$ and as $r \to \infty$.

(ii) $V = f(r)$ on $Z = 0, r \geq 0$.

15. (a) Show that $r \cos\theta$ satisfying the Laplace equations. When r, θ and ϕ are spherical polar co-ordinates.

Or

(b) Determine the temperature $\theta(\rho, t)$ in the infinite cylinder $0 \leq \rho \leq a$ when the initial temperature is $\theta(\rho, 0) = f(p)$ and the surface $\rho = a$ is maintained at zero temperature.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Verify that the equation $z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dx = 0$ is integrable and find its primitive.

17. Prove that the general solution of the linear partial differential equation $P_p + Q_q = R$ is $F(u, v) = 0$, where F is an arbitrary function and $u(x, y, z) = c$, and $v(x, y, z) = c_2$ from a solution of the equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

18. Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

19. (a) Solve the equation $r - s + 2q - z = x^2y^2$.

(b) Find the complementary function of $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

20. Derive D'Alembert's solution of the one–dimensional wave equation.

F–1311

Sub. Code
7MMA2C4

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

MECHANICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a nonholonomic system. Give an example.

2. Define a virtual displacement.

3. State the christoffel symbol of the first kind.

4. What is meant by ignorable coordinates?

5. Define the Hamiltonian function.

6. State the Jacobi's form of the principle of least action.

7. What do you mean by pfaffian differential forms?

8. State the modified Hamilton – Jacobi equation.

9. Define homogeneous canonical transformation.

10. When will you say that the transformation is said to be Mathieu transformation?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove D'Alembert's principle.

Or

(b) Consider a free particle of mass m whose position is given by the Cartesian coordinates (x, y, z). Find the components of generalized momentum.

12. (a) Derive the Lagrange's equations in the standard form for a nonholonomic system.

Or

(b) Two particles, each of mass m are connected by a rigid massless rod of length l. The particles are supported by knife edges placed perpendicular to the rod. Assuming that all motion is confined to the horizontal xy plane, find the Jacobi integral.

13. (a) Discuss the Brachistochrone problem.

Or

(b) Derive the principle of least action.

14. (a) Use Hamilton Jacobi method of analyse the Kepler problem.

Or

(b) Enumerate the following terms:

(i) Hamilton's principle functions;

(ii) Liouville's system.

15. (a) Consider the transformation $Q = \sqrt{2q}e^{t} \cos p$, $P = \sqrt{2q}e^{-t} \sin p$. Show that this transformation is canonical and find the generating function $\phi(q, Q, t)$.

Or

(b) Enumerate the following terms:

(i) Point transformation;

(ii) Momentum transformation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Konig's theorem.

(b) With the usual notations, prove that $\vec{H} = \vec{r}_c + M\dot{\vec{r}}_c + \sum_{i=1}^{N} \vec{P}_i \times m_i \dot{\vec{p}}_i$.

17. A double pendulum consists of two particles suspended by massless rods. Assuming that all motion takes place in a vertical plane, find the differential equations of motion. Linearize these equations, assuming small motions.

18. (a) Define multiplier rule and mention its utility.

(b) Given a holonomic system with a Lagrangian function $L = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2\right) - mgx_3$ and a constant $\dot{x}_1 - \dot{x}_2 + \dot{x}_3 = 0$. Use an augmented Lagrangian function to obtain the differential equations of motion. Solve for $\ddot{x}_1$.

19. Show that the Kepler problem described by the spherical coordinates (r, θ, ϕ) is separable in accordance with the stackel and check Liouville criteria.

20. (a) Derive the Jacobi's identify.

(b) Establish the relationship between Lagrange and Poisson brackets.

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F–1312

Sub. Code
7MMA2E1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective : GRAPH THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions

1. Define a complete graph. Give an example.

2. Define a forest. Give an example.

3. What is meant by cut vertex? Give an example.

4. Define the Hamilton path of G. Give an example.

5. Find the number of different path matching in K_{2n}.

6. What is meant by K–edge–chromatic?

7. Give an examples of a graph such that $\alpha = \beta'$ and $\alpha' = \beta$

8. Draw a 4–critical graphs.

9. State the Jordan curve theorem in the plane.

10. If G is plane graph, then prove that $\sum_{f \in F} d(f) = 2 \in$

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If a K–regular bipartite graph with $K > 0$ has bipartition (X,Y), then prove that $|X| = |Y|$

Or

(b) Prove that every nontrivial loopless connected graph has at least two vertices that are not cut vertices.

12. (a) With the usual notations, prove that $K \leq K' \leq \delta$

Or

(b) Prove that a connected graph has an Euler trial if and only if it has at most two vertices of odd degree.

13. (a) State and prove the Hall's theorem.

Or

(b) Show that the Petersen graph is 4–edge–chromatic.

14. (a) What is meant by independent set of a graph? Give an example. Also prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G

Or

(b) Define the Ramsey numbers. Show that, for all K and l, $r(k,l) = r(l,k)$

15. (a) State and prove Euler's formula for a connected plane graph.

Or

(b) Prove that an inner bridge that avoids every outer bridge is transferable.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, Show that $I(k_n) = n^{n-2}$

17. Prove that a graph G with $V \geq 3$ is 2–connected if and only if any two vertices of G are connected by at least two internally – disjoint paths.

18. If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$

19. State and prove the Brook's theorem.

20. State and prove the five–colour theorem.

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F–1313

Sub. Code
7MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the Hadamard's formula.

2. State the complex form of the Cauchy-Riemann equations.

3. Define a winding number.

4. Compute $\int_{|z|=2} z^n (1-z)^m \, dz$.

5. Distinguish between removable singularity and essential singularity.

6. Find the poles of $\frac{1}{\sin z}$.

7. Find the residues of the function $\frac{e^z}{(z-a)(z-b)}$ at its poles.

8. State the argument principle theorem.

9. Write down the power series expansion of arc $\tan z$.

10. Define an entire function. Give an example.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Show that a harmonic function satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$.

Or

(b) Define a linear transformation. Also prove that a linear transformation carries circle into circles.

12. (a) Prove that the line integral $\int_\gamma p\,dx + q\,dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.

Or

(b) State and prove Liouville's theorem. Also deduce that fundamental theorem of algebra.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the Rouche's theorem.

Or

(b) Evaluate $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}, a>1$.

15. (a) Prove that for $|z|<1, (1+z)(1+z^2)(1+z^4)(1+z^8)\ldots = \frac{1}{1-z}$.

Or

(b) Establish the Jensen's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Abel's theorem.

17. State and prove the Cauchy's representation formula. Deduce that $f^{(n)}(z) = \frac{n!}{2\pi i}\int_C \frac{f(q)dq}{(q-z)^{n+1}}$.

18. State and prove the Schwarz lemma.

19. (a) State and prove the residue theorem.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

20. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} A_n (z-a)^n$ for the function $f(z)$ analytic in $R_1 < |z-a| < R_2$.

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F–1314

Sub. Code
7MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2017 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer ALL questions

1. Define the indiscrete topology. Give an example.
2. Define a cluster point with an example.
3. What is meant by the box topology? Give an example.
4. When will you say that a topological space is said to be matrizable?
5. Define a linear continuum.
6. What are the components and path components of $\mathbb{R}_1$?
7. Is the interval $(0,1)$ compact? Justify your answer.
8. State finite intersection properly.
9. Is in the uniform topology $\mathbb{R}^w$ satisfies the first countability axiom? Justify
10. State the Urysohn lemma.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.

Or

(b) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$

12. (a) Let $\{X_\alpha\}$ be an indexed family of spaces and let $A_\alpha \subset X_\alpha$ for each α. If πX_α is given either the product or the box topology, then prove that $\pi \overline{A}_\alpha = \overline{\pi A_\alpha}$

Or

(b) Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square matrix P are the same as the product topology on $\mathbb{R}^n$.

13. (a) Prove that a finite cartesian product of connected spaces is connected.

Or

(b) Show that a space X is locally connected if and only if for every open set U of X, each component of U is open in X

14. (a) Prove that every compact subspace of a Hausdorff space is closed.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable subcollection covering X.

Or

(b) Define a regular space. Prove that a product of regular spaces is regular.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) If $\mathcal{B}$ is a basics for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $D = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basics for the topology of $X \times Y$

(b) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

17. Let X and Y be topological spaces and let $f : X \to Y$. Prove the following are equivalent.

(a) f is continuous.

(b) For every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$.

(c) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X

(d) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(u) \subset V$.

18. If L is a linear continuum in the order topology, then prove that L is connected and so are intervals and rays in L.

19. Let X be a simply ordered set having the least upper bound property. Prove that in the order topology, each closed interval in X is compact.

20. State and prove the Urysohn metrization theorem.

F–1315

Sub. Code
7MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Third Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Bowl I contains 3 red chips and 7 club chips. Bow II contains 6 red chips and 4 blue chips. A bowl is selected at random and then I chips is draw from this bowl. Compute the probability that this chip is red.

2. Define moment generating function.

3. Define negative distribution.

4. Let $f(x,y)=2 \;\; 0<x<1, 0<y<1$ zero elsewhere. Show that the correlation coefficient of x and y is $p=\frac{1}{2}$.

5. If the m.g.f. of a random variable x is $\left(\frac{1}{2}+\frac{1}{2}e^{t}\right)^{7}$.

6. Let x and y have a bi variate normal distribution with parameters $\mu_1=5, \mu_2=10, \sigma_1^2=1, \sigma_2^2=25$ and $\rho>0$ If $pr(4<y<161x=5)=0.954$. Determine ρ.

7. Determine the constant C in the following so that $f(x)$ is a beta p.d.f. with $f(x)=Cx(1-x)^3, 0<x<1$, zero elsewhere.

8. If the p.d.f. of x is $f(x)=2xe^{-x^2}$, $0<x<\infty$ zero elsewhere determine the p.d.f. of $y=x^2$.

9. Define convergence in distribution.

10. Let y_n have a distribution $b(x,p)$ prove that $1-\frac{y_n}{n}$ converges to $1-p$.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let $f(x)=1, 0<x<1$, zero elsewhere be the p.d.f. of x. find the distribution function and the p.d.f. of $y=(\sqrt{x})$.

Or

(b) Cast a die a number of independent times until a six appears on the up side of the die.

(i) Find the p.d.f. $f(x)$ of x, the number of casts needed to obtain the first six.

(ii) Show that $\sum_{x=1}^{\infty} f(x)=1$.

12. (a) Prove that

(i) $E[E(x_2/x_1)]=E(x_2)$ and

(ii) $Var[E(x_2/x_1)]\leq Var(x_2)$.

Or

(b) Let $f(x_1,x_2)=\frac{1}{16}, x_1=1,2,3,4$ and $x_2=1,2,3,4$, zero elsewhere to be joint p.d.f. of x_1 and x_2. Show that x_1 and x_2 are independent.

13. (a) If the random variable X is $N(\mu,\sigma^2), \sigma^2 > 0$, then prove that the random variable $V = (x-\mu^2)/\sigma^2$ is $\chi^2(1)$.

Or

(b) Find the m.g.f of a binomial distribution. Also find the mean and the variance.

14. (a) Let x have the uniform distribution over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Show that $y = \tan x$ has a cauchy distribution.

Or

(b) Let x_1, x_2 be a random sample from a distribution having the p.d.f. $f(x) = e^{-x}, 0 < x < \infty$, zero elsewhere. Show that $z = \frac{x_1}{x_2}$ has F-distribution.

15. (a) Let $x_1, x_2, \ldots x_n$ be a random sample of size n from a distribution $N(\mu, \sigma^2)$, where $\sigma^2 > 0$. Show that the sum $z_n = \sum_1^n x_i$ does not have a limiting distribution.

Or

(b) Let x_n denote the mean of a random sample of size n from a gamma distribution with the parameters $\alpha = \mu > 0$ and $\beta = 1$. Show that the limiting distribution of $\frac{\sqrt{n}(\overline{X}_n - \mu)}{\sqrt{\overline{X}_n}}$ is $N(0,1)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let x have the p.d.f. $f(x)=(x+2)/18, -2<x<4$, zero elsewhere. Find $E(x), E\left[(x+2)^3\right]$ and $E\left(6x-2(x+2)^3\right)$.

17. Let x and y have the joint p.d.f. described as follows

(x,y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)
$f(x,y)$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

and $f(x,y)$ is equal to zero elsewhere.

(a) Find the means μ_1, μ_2 the variance σ_1^2, σ_2^2 and the correlation coefficient ρ.

(b) Compute $E(y/x=1), E(y/x=2)$, and the line $\mu_2+\rho(\sigma_2/\sigma_2)\,(x-\mu_1)$ do the points $[K, E(y/x=k)], k=1,2$ lie on this line?

18. (a) Compute the measure of skewness and kurtosis of a gamma distribution with parameter α and β.

(b) Show that the graph of a p.d.f. $N(\mu, \sigma^2)$ has pointed of inflection at $x=\mu-\sigma$ and $x=\mu+\sigma$.

19. Derive the p.d.f. of T-distribution.

20. State and prove the central limit theorem.

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F–1316

Sub. Code
7MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Third Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define monoid.

2. Show by an example that the subtraction and division are not binary operations on N.

3. Solve $P(x) = x^5 + 3x^4 - 15x^3 + x - 10$ in telescopic form.

4. Find the characteristic equation of $J(k) - 4J(k-1) + 4J(k-2) = 0$.

5. Write the procedure for finding particular solution.

6. Define recursive function.

7. Let $(<, \leq)$ be a lettice. $P.T\ L$ satisfies communicative law.

8. Draw the Hasse diagram for $D(12)$.

9. Express the polynomial $P(x_1,x_2,x_3) = x_1 \vee x_2$ in an equivalent product of sums canonical form in three variables x_1, x_2 and x_3.

10. Define Karnaugh map for three variables.

Part B $(5 \times 5 = 25)$

Answer **all** the questions choosing either (a) or (b).

11. (a) If g is a homomorphism from a commutative semigroup $(S,*)$ on to a semigroup (T,Δ) is also commutative.

Or

(b) Let $S = N \times N$, N being the set of positive integers and $*$ be an operation on S given by $(a,b)*(c,d) = (a+c, b+d)$. Show that S is a semigroup.

12. (a) Show that $a^n - b^n$ is divisible by $(a-b)$ for all $n \in N$.

Or

(b) Solve $D(k) - 8D(k-1) + 16D(k-2) = 0$ where $D(2) = 16$, $D(3) = 80$.

13. (a) Find the generating function for the following sequences

(i) $S(n) = ba^n$

(ii) $S(n) = n$

(iii) $S(n) = bna^n$.

Or

(b) Show that $f(x,y) = x^y$ is primitive recursive.

14. (a) Let $(L, \leq)$ be a lattice prove that for any $a, b \in L$ the following are equivalent.

(i) $a \leq b$

(ii) $a \vee b = b$

(iii) $a \wedge b = a$.

Or

(b) If $y \leq z$ in L then prove that $x \wedge y \leq x \wedge z$ and $x \vee y \leq x \vee z$ for all $x \in L$.

15. (a) Write down the minterm normal form of $f(x_1, x_2) = \overline{x_1} \vee \overline{x_2}$.

Or

(b) Simplify $f = x_1^1 x_2^1 + x_1 x_3 x_4 + x_1 x_2 x_4^1 + x_2^1 x_3$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $(S_1 *)$ and (T, Δ) be monoids with identities e and e' respectively. Let $g : S \to T$ be an on to (semigroup) homomorphism. Prove that $g(e) = e'$.

17. Solve the recurrence relation $S(k) - 4S(k-1) - 11S(k-2) + 30S(k-3) = 0$, $S(0) = 0, S(1) = -35, S(2) = -85$.

18. If A denotes Ackermann's function evaluate.

(a) $A(3, 1)$

(b) $A(3, 2)$.

19. State and prove the following inequalities

(a) Distributive inequalities

(b) Modular inequalities.

20. For the formula $(P \wedge Q) \vee (\neg R \wedge \neg P)$ draw a corresponding circuit diagram using

(a) NOT, AND and OR gates

(b) NAND gates only.

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F–1317

Sub. Code
7MMA3E4

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Third Semester

Mathematics

Elective – FUZZY MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by normalized fuzzy set?

2. What do you mean by conditional statement?

3. Define sugeno class of fuzzy complement.

4. What do you mean by dual point of a fuzzy set?

5. Define projection of the relation.

6. When will you say that a relation is a normal fuzzy relation?

7. What do you mean by belief measure?

8. Show that $pl(A)+pl(\overline{A})\leq 1$.

9. Prove that $H(x)\geq H(x/y)$.

10. Define conditional information.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Explain the following terms:

(i) Scalar cardinality

(ii) Fuzzy cardinality

Or

(b) In L_3 prove that $\overline{a} \vee \overline{b} \Leftrightarrow \overline{a \wedge b}$.

12. (a) Find the equilibrium point of the sugeno class of complement.

Or

(b) Show that generalized mean defined by

$h_\alpha(a_1, a_2, \ldots a_n) = \left[\dfrac{a_1^\alpha + a_2^\alpha + \ldots + a_n^\alpha}{n}\right]^{1/\alpha}$ become min and max operation for $\alpha \to -\infty$ and $\alpha \to \infty$ respectively.

13. (a) The fuzzy relation R defined on sets $X_1 = \{a, b, c\}$, $X_2 = \{s, t\}$, $X_3 = \{x, y\}$ and $X_4 = \{i, j\}$ as follows.

$$R(x_1, x_2, x_3, x_4) = \frac{0.4}{b,t,y,i} + \frac{0.6}{a,s,x,i} + \frac{0.9}{b,s,y,i} + \frac{1}{b,s,y,j} + \frac{0.6}{a,t,y,j} + \frac{0.2}{c,s,y,i}$$

Compute the projections $R_{1.2.4}$.

Or

(b) Find $\mu_{p \odot Q}$ where

$$\mu_P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{pmatrix} \end{array} \text{ and } \mu_Q = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cccc} \alpha & \beta & \gamma & \delta \end{array} \\ \begin{pmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{pmatrix} \end{array}$$

14. (a) Let $X = \{a,b,c,d\}$ be the universal set giving the basic assignment $m(\{a,b,c\}) = 0.5$, $m(\{a,b,d\}) = 0.2$, $m(x) = 0.3$. Determine the corresponding plausibility measure.

Or

(b) Show that every possibility measure can be uniquely determine by a possibility distribution function.

15. (a) Consider two fuzzy sets, A and B defined on the set of real numbers $x = [0,4]$ by the membership grad function $\mu_A(x) = \frac{1}{1+x}$ and $\mu_B(x) = \frac{1}{1+x^2}$. Draw a graph for these functions.

Or

(b) Enumerate the following terms:

(i) Shannon entropy

(ii) Normalized Shannon entropy

(iii) Measure of confusion.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that all α-cuts of any fuzzy set A defined on $\mathbb{R}^n$, $n \geq 1$ are convex if and only if $\mu_A(\lambda r + (1-\lambda)s) \geq \min\{\mu_A(r), \mu_A(s)\}$, $r_1 s \in \mathbb{R}^n$, $\lambda \in [0,1]$.

17. Prove that $\lim_{w \to \infty} i_w(a,b) = \min(a,b)$.

18. Solve the following fuzzy relation equation.

$$P \circ \begin{bmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.6 & 0.5 \end{bmatrix}$$

19. Prove that

(a) Bel $(A \cap B) = \min\{Bel(A), Bel(B)\}\ \forall A, B \in \mathcal{P}(x)$

(b) Pl $(A \cup B) = \max\{pl(A), pl(B)\} \forall A, B \in \mathcal{P}(x)$.

20. Let m_x and m_y be marginal basic assignments on set X and Y respectively and let m be a joint basic assignment on $x \times y$ such that $m(A \times B) = m_x(A).m_y(B)$ for all $A \in \mathcal{P}(x)$ and $B \in \mathcal{P}(y)$. Prove that $E(m) = E(m_x) + E(m_y)$.

F–1318

Sub. Code
7MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define normed space.

2. What do you mean by Hilbert cube?

3. State Hahn Banach extension theorem.

4. What is meant by Banach space?

5. What do you mean by projection map?

6. State closed graph theorem.

7. Define normed dual space.

8. Stare Hausdorff theorem.

9. State polarization identity.

10. Define a Hilbert space.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let X be a normed space, Y be a closed subspace of X and $Y \neq X$. Let r be a real number such that $0 < r < j$ Prove that there exists some $x_r \in X$ such that $\|x_r\| = 1$ and $r <$ dist $(x_r, y) \leq 1$.

Or

(b) Let X and Y be normed spaces. If X is finite dimensional, then prove that every linear map from X to Y is continuous.

12. (a) State and prove Hahn- Banach separation theorem.

Or

(b) A normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

13. (a) State and prove uniform boundedness principle.

Or

(b) Let X be a linear space over K. consider subsets U and V of X, and $k \in K$ such that $U \subset V + kU$ prove that for every $x \in U$, there is a sequence (v_n) in V such that $x - \{v_1 + kv_2 + ... + k^{n-1}v_n\} \in k^n U,\ n = 1,2.....$

14. (a) Let X be a normed space, if X^1 is separable then prove that X is separable.

Or

(b) Let X,Y, and Z be normed spaces. Let F_1 and F_2 be in $BL(X,Y)$ and $k \in K$. Prove that $(F_1 + F_2)^1 = F_2^1 + F_2^1$; $(kF_1^1) = kF_1^1$. Also let $F \in BL(X,Y)$ and $G \in BL(Y,Z)$ prove that $(GF)^1 = F^1G^1$.

15. (a) State and prove Schwarz inequality.

Or

(b) State and prove Bessel's inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Ascoli's lemma.

17. State and prove Taylor- Foguel theorem.

18. State and prove open mapping theorem.

19. State and prove Riesz representation theorem for L^P.

20. State and prove unique Hahn- Banach extension theorem.

F–1319

Sub. Code
7MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a cut and the cut capacity in a network.
2. Define the total float and the free float.
3. Define holding cost.
4. What is meant by setup cost?
5. Define exponential distribution.
6. What is forgetfulness property?
7. Write down the Little's formula.
8. Write down the formula for finding Ls in

   $(M/G/1):(GD/\infty/\infty)$ (P-k formula) model.

9. When will you say that the function $f(x_1, x_2, \ldots, x_n)$ is separable?

10. Write a short note on steepest ascent method.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Enumerate the maximal flow model.

Or

(b) Construct the network diagram comprising activities $B, C, \cdots, Q$ and N such that the following constraints are satisfied :

$B < E, F; C < G, L : E, G < H; L, H < I; L < M; H < N$,
$H < J; I, J < P; P < Q$.

The notation $X < Y$ means that the activity X must be finished before Y can begin.

12. (a) Lube can specialize in fast automobile oil change. The garbage buys can oil in bulk at $3 per gallon. A discount price of $2.50 per gallon is available if Lube can purchases more than 1000 gallons. The garage services approximately 150 cars per day and each oil change takes 1.25 gallons. Lube can stores bulk oil at the cost of $.02 per gallon per day. Also, the cost of placing an order for bulk oil is $20. There is a 2 day lead time for delivery. Determine the optimal inventory policy.

Or

(b) Describe the classic EOQ model.

13. (a) Narrate the pure death model.

Or

(b) Babies are born on a sparsely populated state at the rate of one birth every 12 minutes, the time between births follows an exponential distribution. Determine the following:

(i) The average number of births per year.

(ii) The probability that no births will occur in any one day.

14. (a) For $(M/M/1) : (GD/N/\infty)$ queueing model. Show that the two expressions for λ_{eff} are equivalent namely $\lambda_{eff} = \lambda(1 - P_N) = \mu(L_s - L_q)$.

Or

(b) In a railway marshalling yard, goods train arrive at a rate of 30 trains per day. Assuming that the interarrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average of 36 minutes.

Find the following:

(i) The average number of trains in the queue.

(ii) The probability that the queue size exceeds 10.

15. (a) Find the maximum of the following function by Gradient method:

$$f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2 .$$

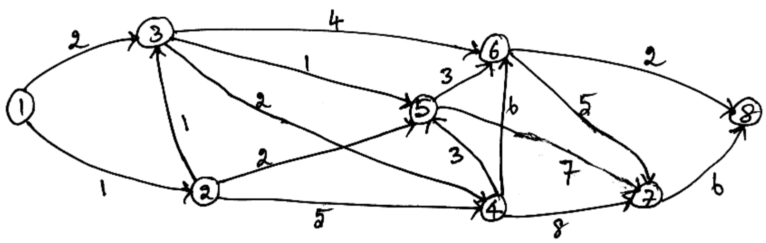
Or

(b) Enumerate the constrained algorithm.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Use Dijkstra's algorithm to find the shortest path from source 'a' to destination 'f' from the following network:



17. The following table provides the data for a 3-period inventory situation.

Period i	Demand D_i (units)	Setup cost k_i ($)	Holding cost hi *($)*
1	3	3	1
2	2	7	3
3	4	6	2

The demand occurs in descrete units, and the starting inventory is $x_1 = 1$ unit. The unit production cost is \$10 for the first 3 units and \$20 for each additional units, which is translated mathematically as

$$C_i(z_i) = \begin{cases} 10z_i & 0 < z_i \le 3 \\ 30 + 20(z_i - 3) & z_i \ge 4 \end{cases}$$

Find the optimal inventory policy.

18. The florist section in a grocery store stocks 18 dozen roses at the beginning of each week. On the average, the florist sells 3 dozens a day (one dozen at a time), but the actual demand follows a Poisson distribution. Whenever the Stock level reaches 5 dozens, a new order of 18 new dozens is placed for delivery at the beginning of the following week. Because of the nature of the item, all roses left at the end of the week are disposed of:

    Determine the following:

    (a) The probability of placing an order in any one day of the week.

    (b) The average number of dozen roses that will be discarded at the end of the week.

19. Derive $L_q, L_s, W_q . W_s$ for $(M/M/C) : (GD/\infty/\infty)$ queueing model.

20. Use separable convex programming to solve the NLPP.

Maximize $z = x_1 - x_2$

Subject to : $3x_1^4 + x_2 \le 243$

$x_1 + 2x_2^2 \le 32$

$x_1 \ge 2.1$

$x_2 \ge 3.5$

F–1320

Sub. Code
7MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

TOPOLOGY — II

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the one-point compactification. Give an example.

2. Is the rationals Q locally compact? Justify your answer.

3. What is meant by completely regular space?

4. Define the stone-cech compactification.

5. Prove that the collection $\mathcal{A} = \left\{ \frac{n, n+2}{n \in z} \right\}$ is locally finite.

6. Define a G_S - set. Give an example.

7. When will you say that the matrix space is said to be totally bounded?

8. Define an equicontinuous.

9. What is meant by the evaluation map?

10. Define a Baire Space with an example.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Show that $[0,1]^{w}$ is not locally compact in the uniform topology.

Or

(b) Let x be a set and let $\mathcal{D}$ be a collection of subsets of x that is maximal with respect to the finite intersection property. If A is a subset of x that intersects every element of $\mathcal{D}$, then prove that A is an element of $\mathcal{D}$.

12. (a) Show that every locally compact Hausdorff space is completely regular.

Or

(b) Let $A \subset x$ and let $f : A \to z$ be a continuous map of A into the Hausdorff space z. Prove that there is at most one extension of f to a continuous function $g : \overline{A} \to z$.

13. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of x. Prove that the collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

Or

(b) Find a nondiscrete space that has a countably locally finite basis but does not have a countable basis.

14. (a) Define a Cauchy sequence. Also prove that a metric space x is complete if every Caushy sequence in x has a convergent subsequence.

Or

(b) If x is locally compact or if x satisfies the first countability axiom, then prove that x is compactly generated.

15. (a) Let x is locally compact Hausdorff space and let $e(x,y)$ have the compact open topology. Prove that the map $e: x \times e(x,y) \to y$ defined by the equation $e(x,f) = f(x)$ is continuous.

Or

(b) State and prove the Baire category theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Tychonoff theorem.

17. Let x be a completely regular space. Prove that there exists a compactification y of x having the property that every bounded continuous map $f: x \to \mathbb{R}$ extends uniquely to a continuous map of y into $\mathbb{R}$.

18. Let x be a metrizable space. If $\mathcal{A}$ is an open covering of x, then prove that there is an open covering ε of x refining $\mathcal{A}$ that is countably locally finite.

19. Let $I = [0,1]$. Prove that there exists a continuous map $f: I \to I^2$ whose image fills up the entire square I^2.

20. State and prove the Ascoli's theorem.

F–1321

Sub. Code
7MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

Elective – ADVANCED STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the confidence interval for the when σ is known in $N\left(\mu, \sigma^2\right)$.

2. Define statistical hypothesis and likelihood function.

3. Define the minimum mean - square - error estimator.

4. Define a complete family of probability density functions.

5. Write a short notes on Bayesian estimation.

6. What is meant by asymptotically efficient?

7. Define a best critical region of size α.

8. Define a non central t-distribution.

9. State the assumptions of analysis of variance.

10. Write a short notes on correlation coefficient of a random sample.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let $x_1, x_2, ..., x_n$ represent a random sample from each of the distribution having p.d.f. $f(x;\theta) = \theta^x e^{-\theta} / x!$, $x = 0, 1, 2, ...,$ $0 \leq \theta < \infty$ Zero elsewhere, where $f(\theta;0) = 1$. Find the maximum likelihood estimator $\hat{\theta}$ of θ.

Or

(b) Let a random sample of size 17 from the normal distribution $N(\mu, \sigma^2)$ yield $\bar{x} = 4.7$ and $s^2 = 5.76$. Determine a 90% confidence interval for μ.

12. (a) Prove that n[th] order statistic of a random sample of size n from a uniform distribution with p.d.f. $f(x;\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x < \theta, \quad 0 < \theta < \infty \\ 0, & \text{elsewhere} \end{cases}$ is a sufficient statistic for θ.

Or

(b) Let $x_1, x_2, ...x_n$ denote a random sample from a distribution that is $N(\theta, 1), -\infty < \theta < \infty$. Find the unbiased minimum variance estimator of θ^2.

13. (a) Let $x_1, x_2, ...x_n$ denote a random sample from a distribution which is $b(1, \theta), 0 < \theta < 1$. Determine the decision functions δ which is a Baye's solution.

Or

(b) Given the p.d.f. of Cauchy distribution $f(x;\theta) = \frac{1}{\pi\left[1+(x-\theta)^2\right]}, -\infty < x < \infty, -\infty < \theta < \infty$.

Prove that the Cramer-Rao lower bound is $\frac{2}{n}$, where x is the sample size.

14. (a) Enumerate the uniformly most powerful test.

Or

(b) Let X be $N(\theta, 100)$. Find the sequential probability ratio test for testing $H_0 : \theta = 75$ aganist $H_1 : \theta = 78$.

15. (a) Derive non central F distribution.

Or

(b) State and prove the Boole's inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Narrate the following test for

(a) Goodness of fit and

(b) Independence of attributes rising Chi-square test.

17. State and prove the Neyman factorization theorem for the existence of a sufficient statistics for a parameter.

18. (a) State and prove the Rao-Cramer inequality.

(b) Discuss the limiting distribution of maximum likelihood estimator.

19. A random sample $x_1, x_2, ...x_n$ arises from a distribution given by $H_0 : f(x;\theta) = \frac{1}{\theta}, 0 < x < \theta,$ zero elsewhere, or $H_1 : f(x;\theta) = \frac{1}{\theta} e^{-xy\theta}, 0 < x < \infty,$ zero elsewhere. Determine the likelihood ratio (λ) test associated with the test of H_0 against H_1.

20. With the usual notations, prove the following:

(a) $Q = Q_3 + Q_4$

(b) $$\sum_{i=1}^{n} [y_i - \alpha - \beta (x_i - \bar{x})]^2 = n (\hat{\alpha} - \alpha)^2 + (\hat{\beta} - \beta)^2$$

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 + \sum_{i=1}^{n} [y_i - \hat{\alpha} - \hat{\beta} (x_i - \bar{x})]^2 .$$

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F–1322

Sub. Code
7MMA4E3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

Elective – NUMERICAL METHODS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define order and convergence.

2. State sturm's theorem.

3. Determine the Euclidean and the maximum absolute row sum norms of the matrix $A = \begin{pmatrix} 1 & 7 & -4 \\ 4 & -4 & 9 \\ 12 & -1 & 3 \end{pmatrix}$.

4. If A is a strictly diagonally dominant matrix, then prove that the Jacobi iteration scheme converges for any initial strating vector.

5. List the disadvantages of Quadratic splines.

6. State Weierstrass approximation theorem.

7. What is mean by truncation error and round off error in Numerical differentiation?

8. Define the order of integration method.

9. Define (a) absolutely stable (b) Relatively stable

10. What is (a) explicit method (b) Implicit method?

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) How should the constant α be chosen to ensure the fastest possible convergence with the iteration formula, $x_{n+1} = \dfrac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ form the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use the initial approximation $P_0 = 0.5, q_0 = 0.5$.

12. (a) Determine the condition number of the matrix $A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{pmatrix}$ using the maximum absolute row sum norm.

Or

(b) For the matirx $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

(i) Find the eigen values and the corresponding eigen vectors.

(ii) Verity that S^{-1} is a diagonal matrix, where S is the matrix of eigen vectors.

13. (a) Determine the parameters in the formula $P_{(x)} = a_0(x-a)^3 + a_1(x-a)^2 + a_2(x-a) + a_3$

such that $P(a) = f(a), P'(a) = f'(a)$

$$P(b) = f(b),\ P'(b) = f'(b).$$

Or

(b) Obtain the least squares polynomial approximation of degree one and two for $f(x) = x^{\frac{1}{2}}$ on $[0,1]$.

14. (a) Find the Jacobian matrix for the system of equations $f_1(x,y) = x^3 + xy^2 - y^3 = 0$ $f_2(x,y) = xy + 5x + 6y = 0$ at the point $(1,2)$.

Or

(b) Evaluate the integral $I = \int_{-1}^{1} (1-x^2)^{3/2} \cos x \, dx$ using the Gauss-chebyshev 1-point, 2 piont and 3- point quadrature rules. Evaluate it also using the Gauss-Legendre 3-point formula.

15. (a) Solve the initial value problem $u' = -2tu^2, u(0) = 1$ using mid-point method with $h = 0.2$ over the interval $(0,1)$.

Or

(b) Find the three term Taylor series solution for the third order initial value problem.

$W''' + WW'' = 0,\ W(0) = 0$

$W'(0) = 0, W''(0) = 1$

Find the bound on the error for $t \in [0, 0.2]$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Apply Graffe's root squaring method to find the roots of the following equation correct to two decimal places: $x^3 - 2x + 2 = 0$.

17. Find all the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$ using the Rutishauser method. Iterate till the elements of the lower triangular part are less than 0.05 in magnitude.

18. Give the following values of $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

Estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

19. Assume that $f(x)$ has a minimum in the interval $x_{n-1} \le x \le x_{n+1}$ where $x_k = x_0 + kh$. Show that the interpolation of $f(x)$ by a polynomial of second degree yields the approximation

$$f_n - \frac{1}{8}\left[\frac{(f_{n+1} - f_{n-1})^2}{f_{n+1} - 2f_n + f_{n-1}}\right] \ (f_k = f(x_k)).$$

20. Solve the initial value problem.

$u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0,0.4]$. use the second order implicit Runge - Kutta method.